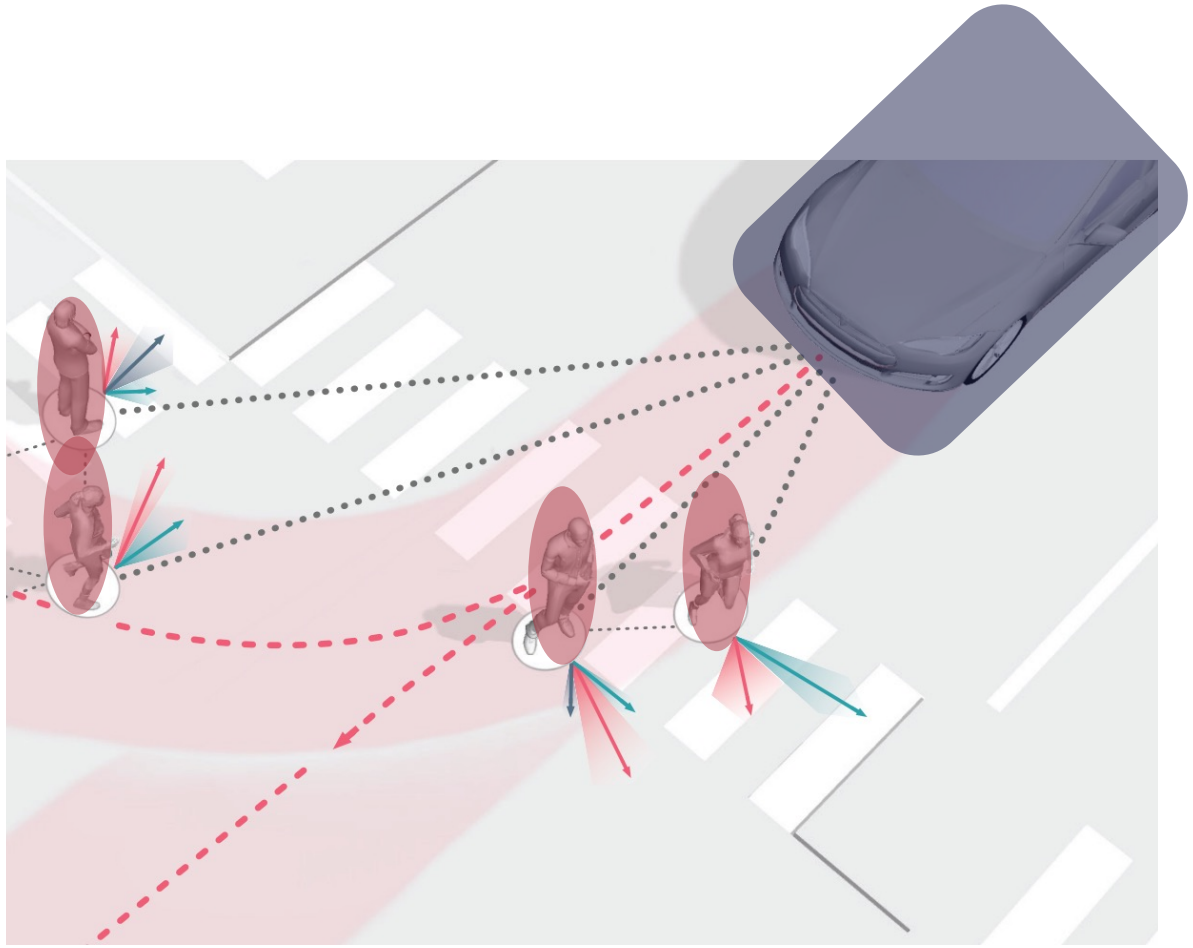
KI WISSEN
Automotive AI Powered by Knowledge

KI Wissen Final Event | 21-22 March 2024

Model Predictive Control under Temporal Logic Specifications

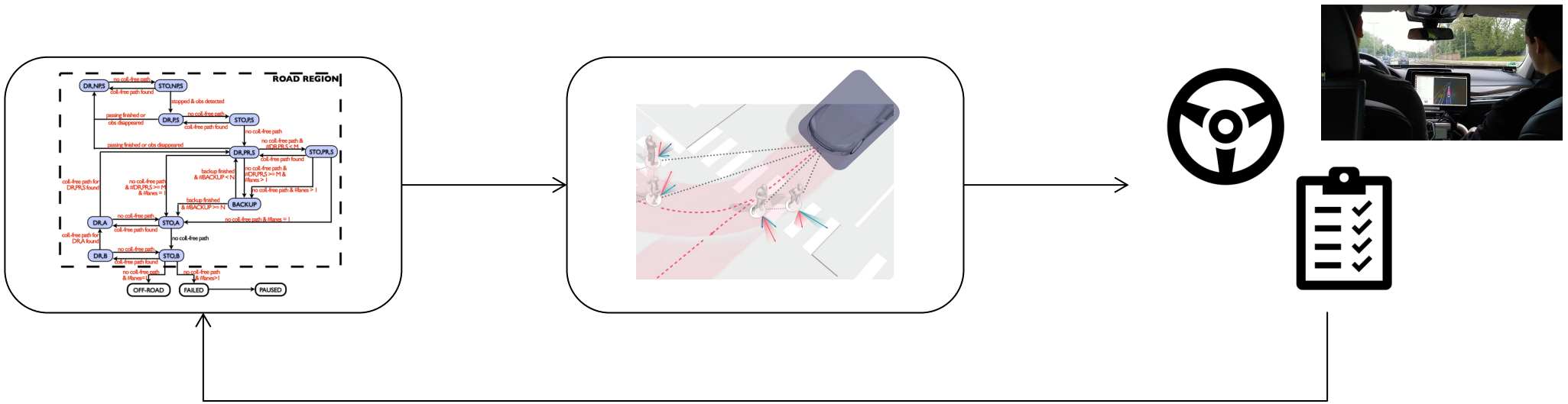
Etienne Bührle | FZI

Motivation



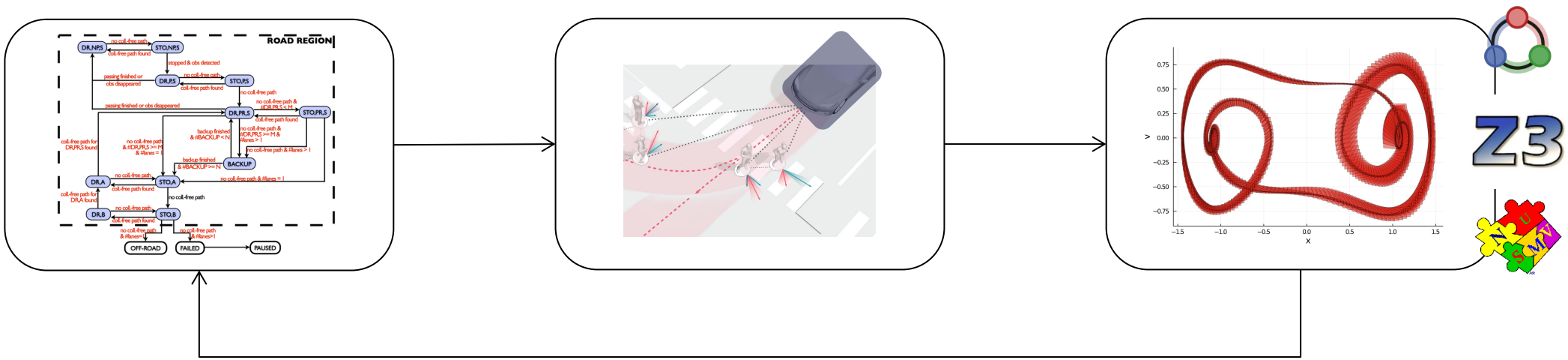
Salzmann et al.

Verification



Salzmann et al.; Wongpiromsarn; Prism; Z3; nuSMV

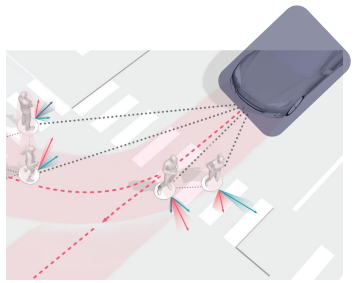
Verification

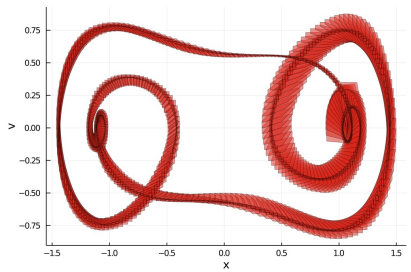


Salzmann et al.; Wongpiromsarn; JuliaReach; Prism; Z3; nuSMV

Verification

Can we synthesize controllers from specifications?

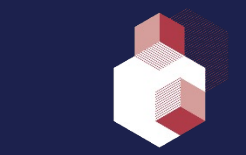




Salzmann et al.; Wongpiromsarn; JuliaReach; Prism; Z3; nuSMV

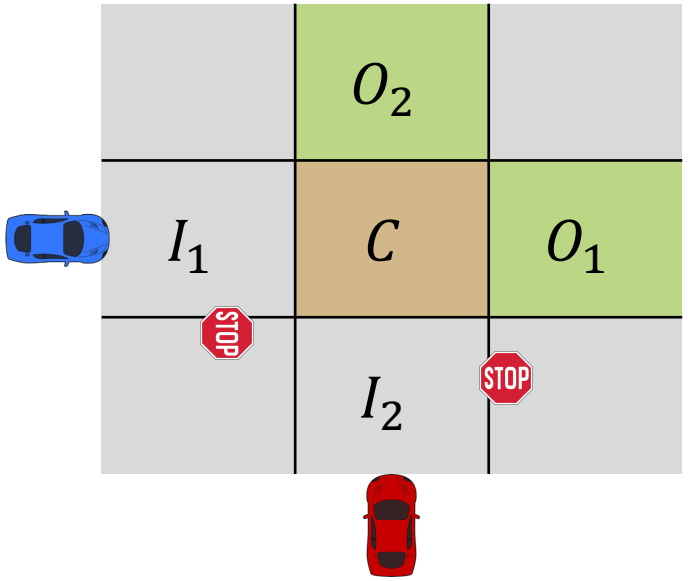
Agenda

1. Formal Synthesis for Linear Systems
2. Multiagent Control
3. Control Barrier Safety Layer



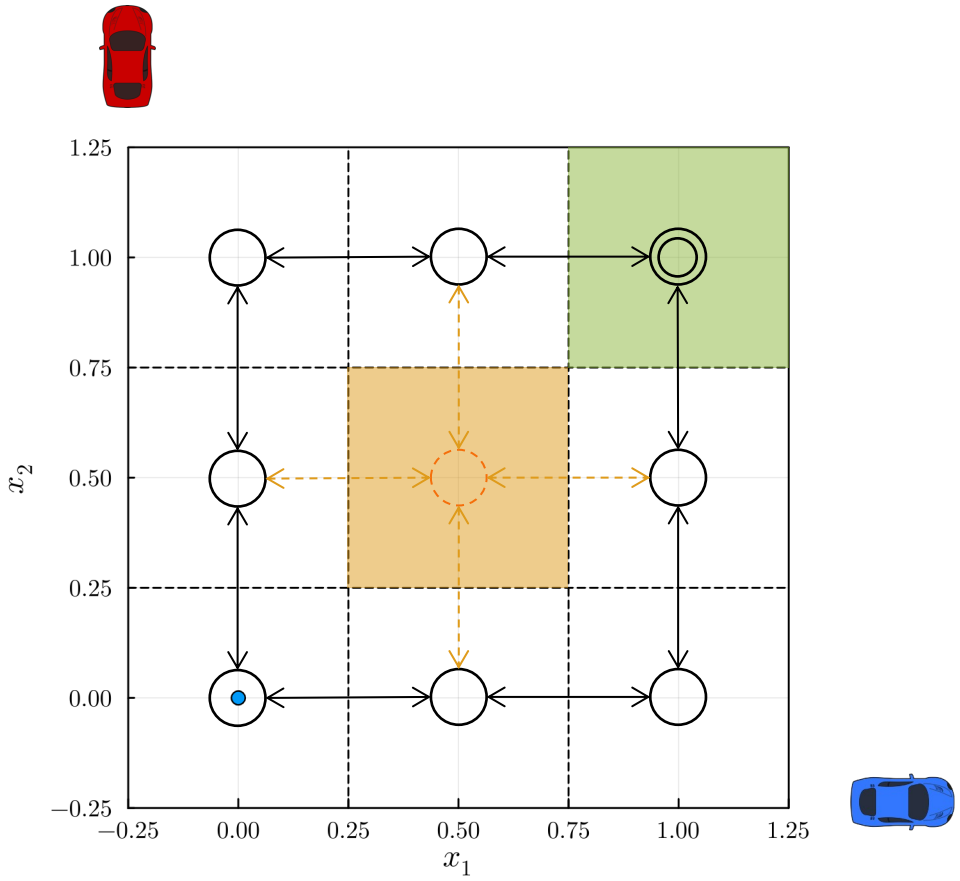
Formal Synthesis for Linear Systems

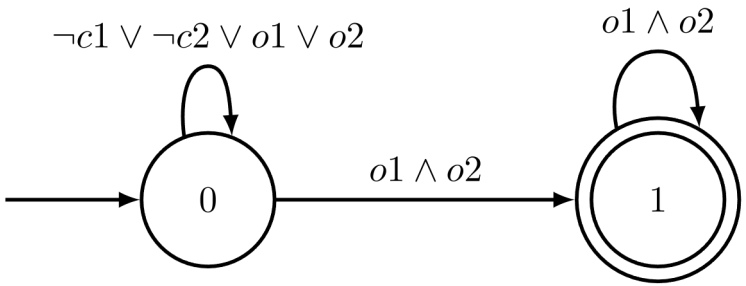
Formal Synthesis for Linear Systems



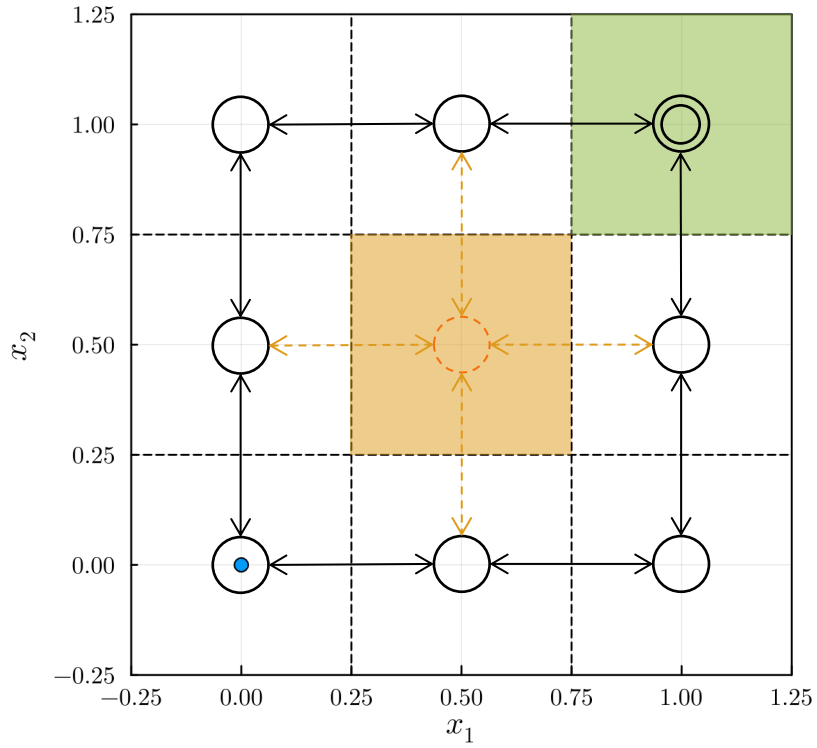
$$G\neg(x_1 \in C \wedge x_2 \in C)$$
$$\wedge F(x_1 \in O_1)$$
$$\wedge F(x_2 \in O_2)$$

Formal Synthesis for Linear Systems





Quasi-Infinite Horizon OCP



$$\min_{x,u,q} \int_0^T c(x,u)dt + V(x(T))$$

$$s.t. \quad x(T) \in \mathcal{T} \qquad D(q) \leq 0$$

$$C(x,u) = 0 \qquad S(q,x) \leq 0$$

Value Function

Moment LP

$$\min_{\mu,\mu_T} \langle l, \mu \rangle$$

$$\text{s.t.} \quad div(f\mu) + \mu_T = \mu_0$$

$$\mu_0, \mu_T, \mu \in M(X \times U)$$

Dual Moment LP

$$\max_{v} \langle v, \mu_0 \rangle$$

$$\text{s.t.} \quad l + \nabla v \cdot f \in C(X \times U)$$

$$l_T - v \in C(X_T)$$

Positivity constraints

Value Function

Moment LP

$$\min_{\mu,\mu_T} \langle l, \mu \rangle$$

$$\text{s.t.} \quad div(f\mu) + \mu_T = \mu_0$$

$$\mu_0, \mu_T, \mu \in M(X \times U)$$

Dual LP

$$\max_{v} \langle v, \mu_0 \rangle$$

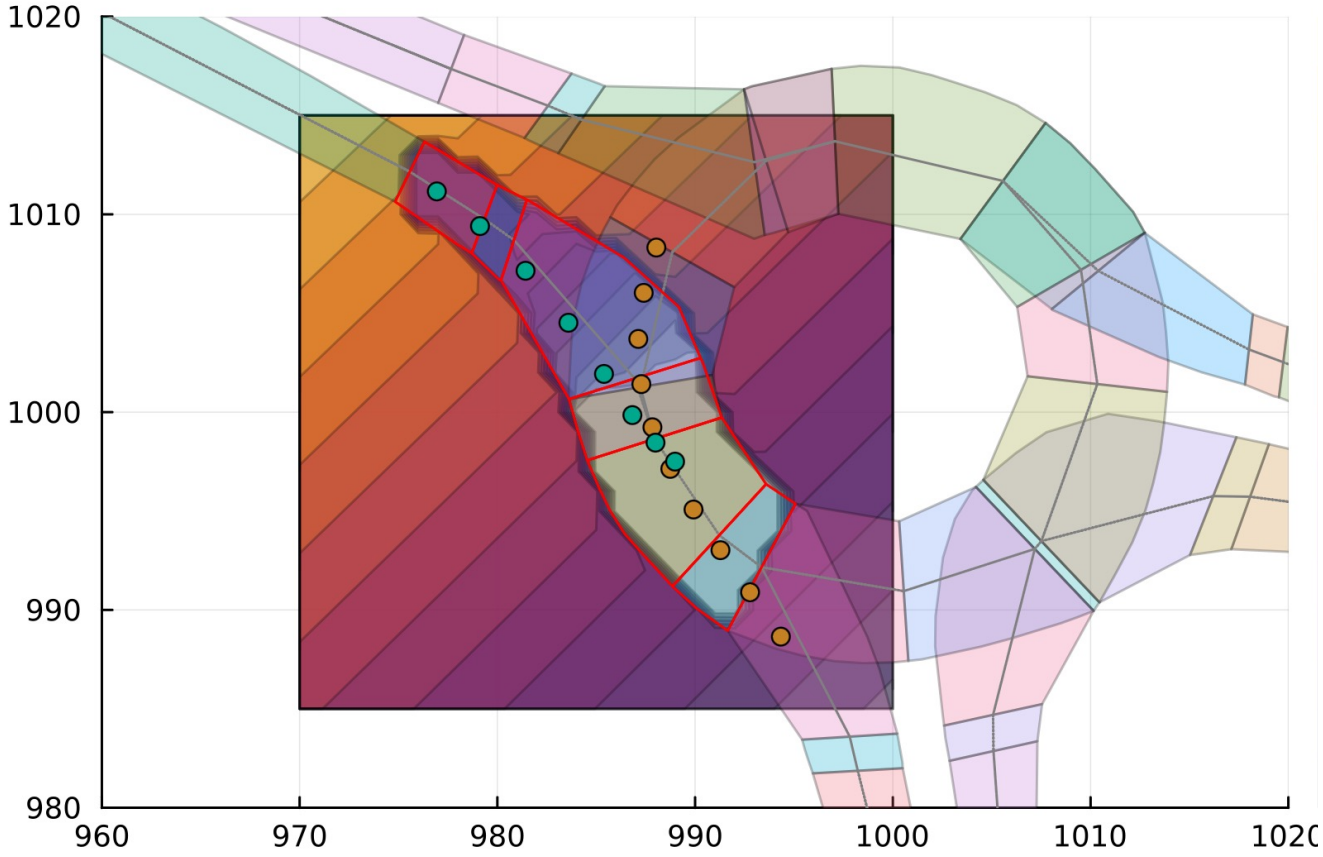
Approximate via sampling

$$\text{s.t.} \quad l + \nabla v \cdot f \geq 0 \quad \forall x, u \in S(X \times U)$$

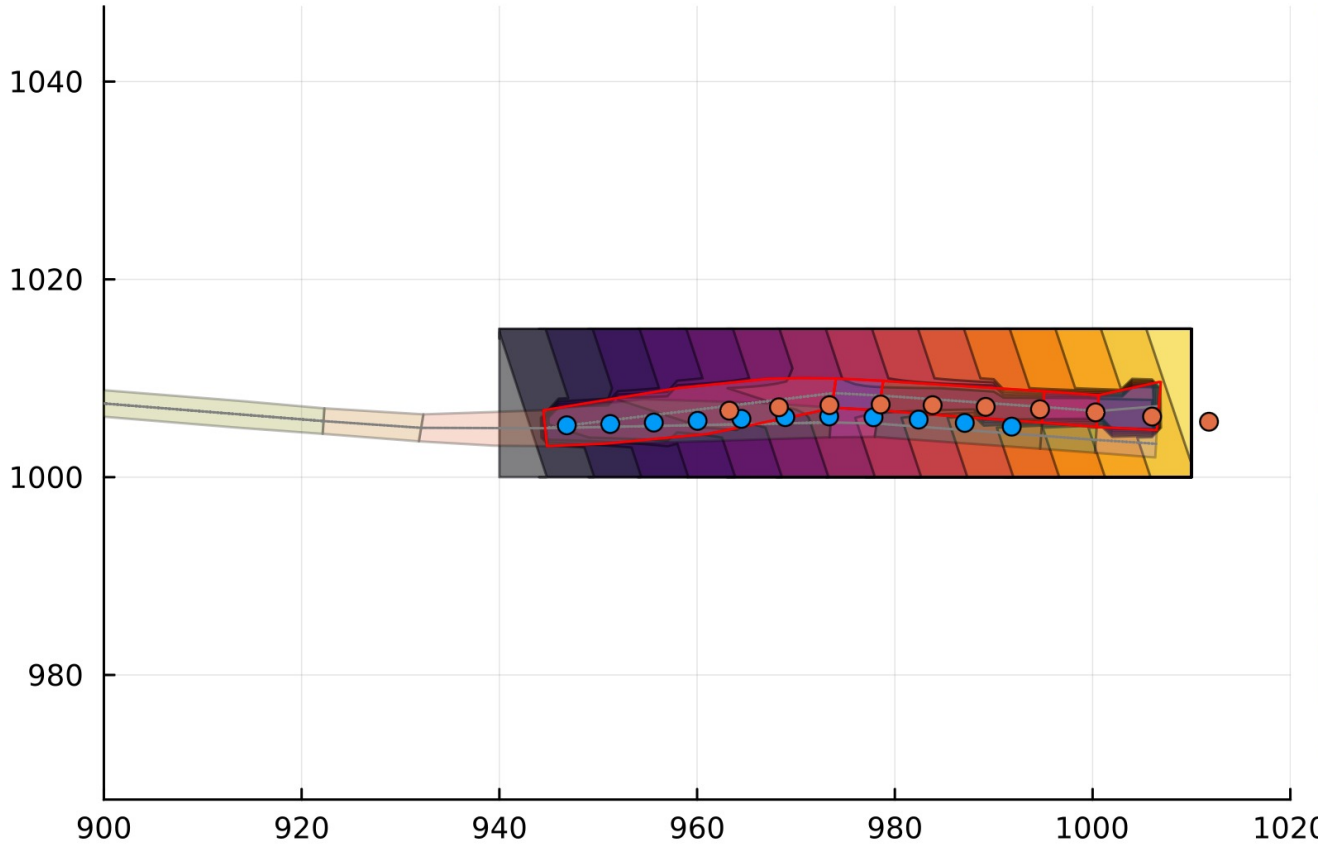
$$l_T - v \geq 0 \quad \forall x \in C(X_T)$$

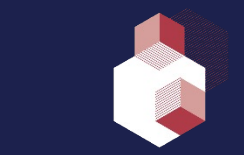
=> Linear Program

Results



Results





Multiagent Control

Method

Optimize individually

$$\min_{x,y,q} J_x(x,y) + \lambda_1(y - y^0) \qquad \min_{y,x,q} J_y(y,x) + \lambda_2(x - x^0)$$

$$s.t.\ S_x(x,y) = 0 \qquad s.t.\ S_y(y,x) = 0$$

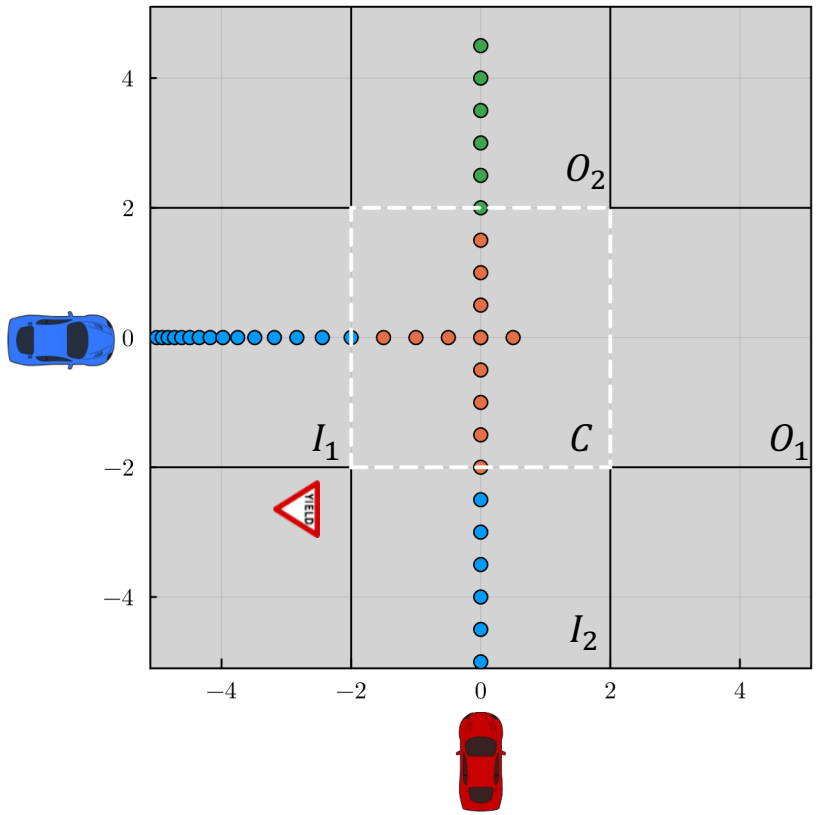
Penalize deviation of shared variables

$$\lambda_1' = \lambda_1 + \alpha(y - y^0) \qquad \lambda_2' = \lambda_2 + \alpha(x - x^0)$$

Update shared variables, iterate

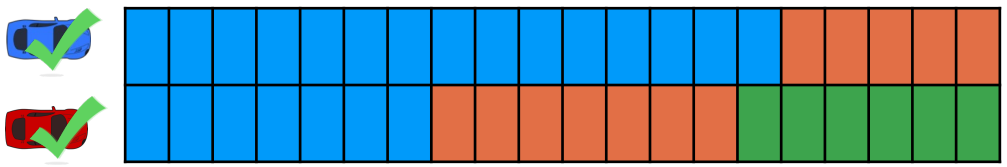
Update y^0 Update x^0

Example

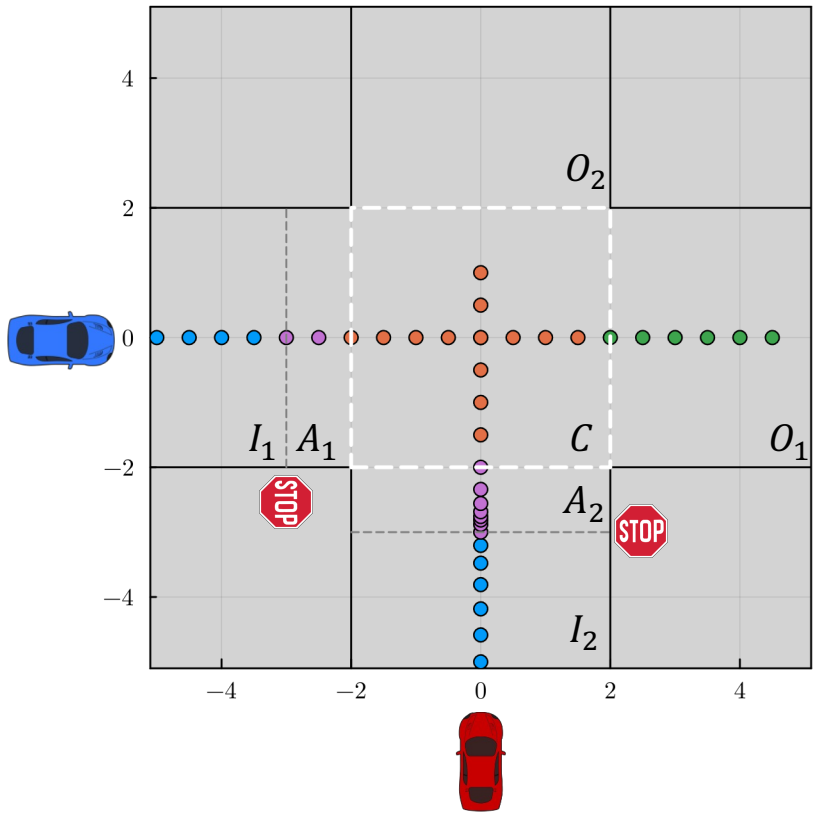


$G\neg(x_1 \in C \wedge x_2 \in C)$

$true$

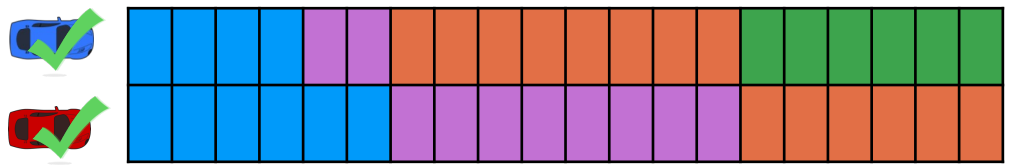


Example



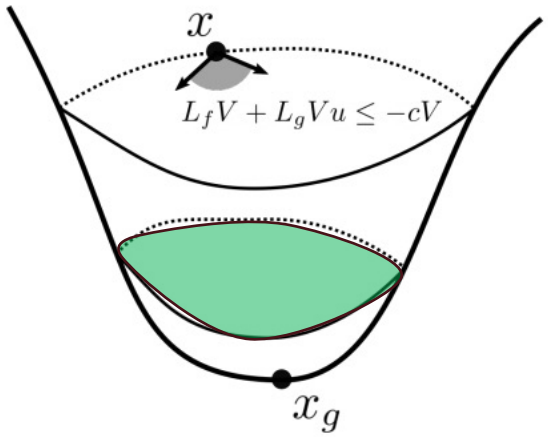
$G((x_1 \in I_1 \wedge x_2 \in A_2) \Rightarrow G\neg(x_1 \in C \wedge x_2 \in C))$

$G((x_2 \in I_2 \wedge x_1 \in A_1) \Rightarrow G\neg(x_2 \in C \wedge x_1 \in C))$



Control Barrier Safety Layer

Control Barrier Functions



- Given system dynamics $\dot{x} = f(x, u)$ and barrier function

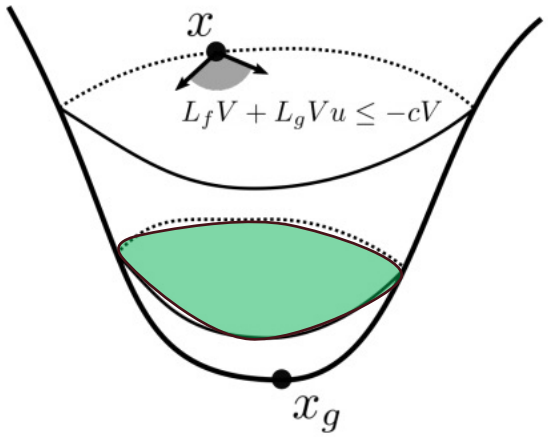
$$h(x) \leq 0, \qquad x \in X_{safe}$$
$$h(x) > 0, \qquad x \notin X_{safe}$$

- We want the system trajectories to stay inside the safe set

$$\dot{h} = \nabla_x h \cdot f(x, u) \leq -c \cdot h(x)$$

Gao et al.

Control Barrier Functions



- Given system dynamics $\dot{x} = f(x, u)$ and barrier function

$$h(x) \leq 0, \qquad x \in X_{safe}$$
$$h(x) > 0, \qquad x \notin X_{safe}$$

- For input-affine systems, these constraints are affine in u!

$$\nabla_x h \cdot (f(x) + g(x) \cdot u) \leq -c \cdot h(x)$$

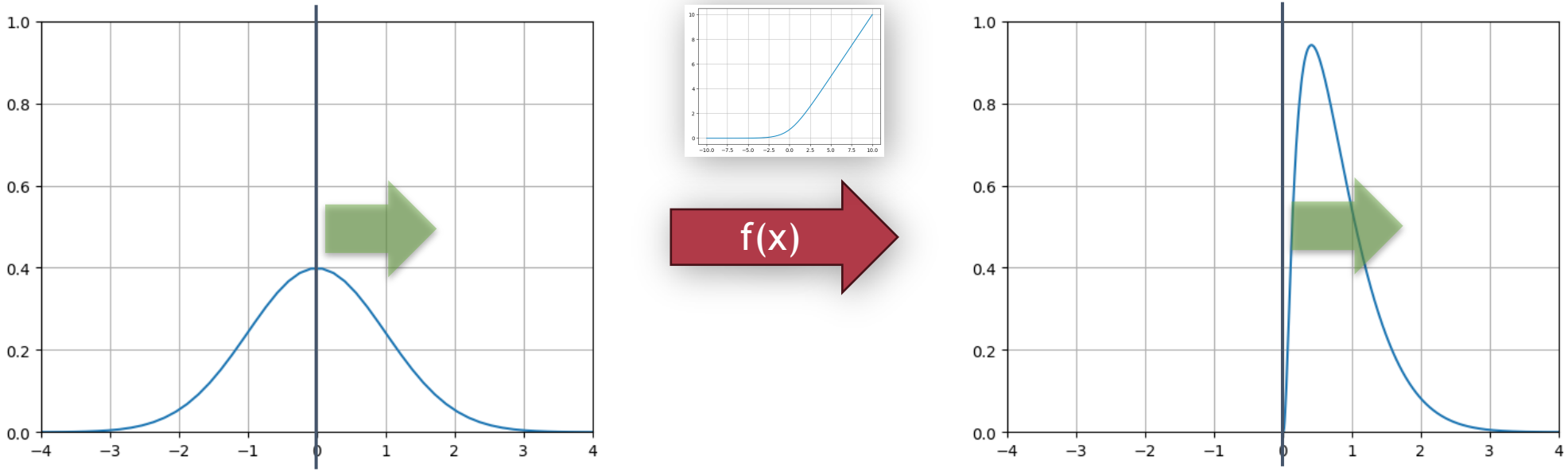
Gao et al.

Projection Layer

- Project density into safe set using change of variables

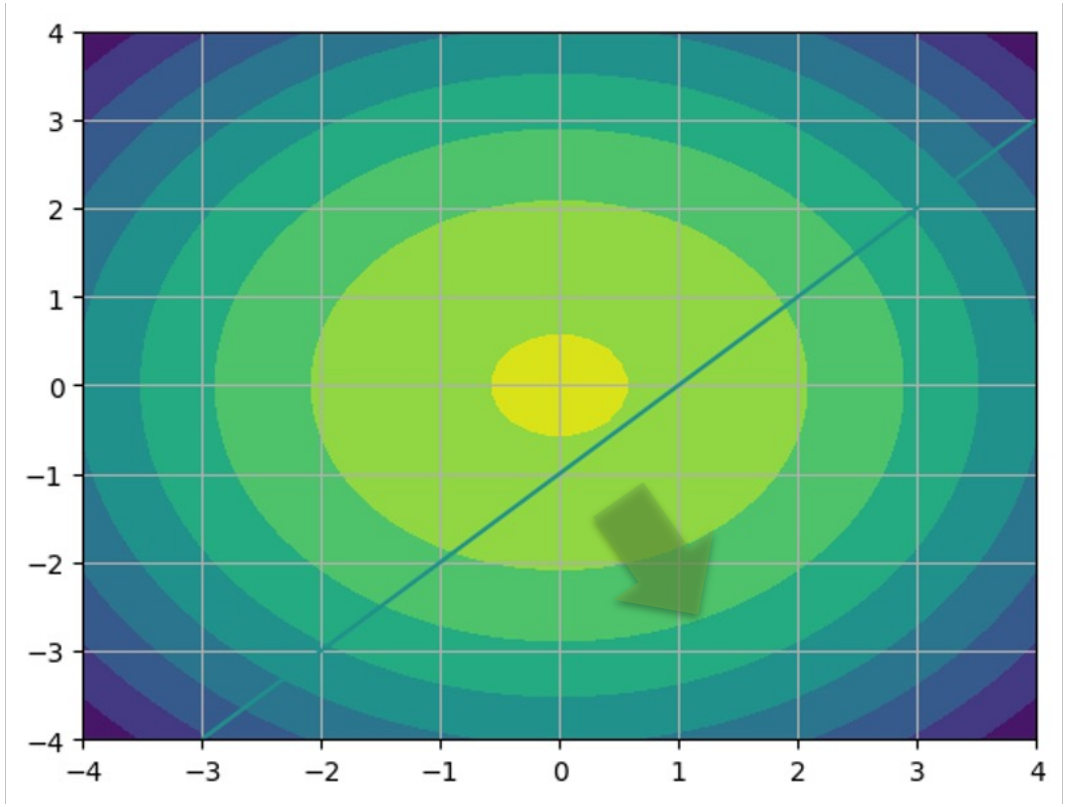
$$p_x(x)$$

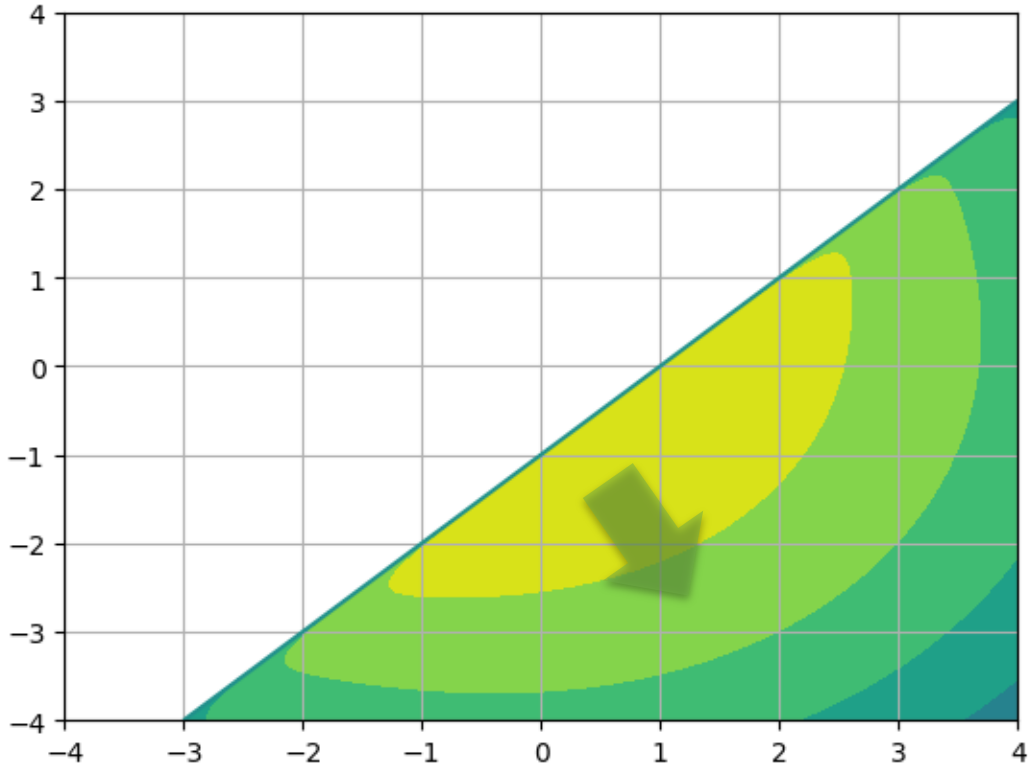
$$p_z(z) = p_x\left(f^{-1}(z)\right)\left|\det\frac{\partial f}{\partial x}\right|$$



Projection Layer

- Project density into safe set using change of variables





Vanilla Policy Gradient

Algorithm 1 Vanilla Policy Gradient Algorithm

1: Input: initial policy parameters θ_0, initial value function parameters ϕ_0
2: **for** $k = 0, 1, 2, \ldots$ **do**
3: Collect set of trajectories $\mathcal{D}_k = \{\tau_i\}$ by running policy $\pi_k = \pi(\theta_k)$ in the environment.
4: Compute rewards-to-go $\hat{R}_t$.
5: Compute advantage estimates, $\hat{A}_t$ (using any method of advantage estimation) based on the current value function V_{ϕ_k}.
6: Estimate policy gradient as

$$\hat{g}_k = \frac{1}{|\mathcal{D}_k|} \sum_{\tau \in \mathcal{D}_k} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t)|_{\theta_k} \hat{A}_t.$$

7: Compute policy update, either using standard gradient ascent,

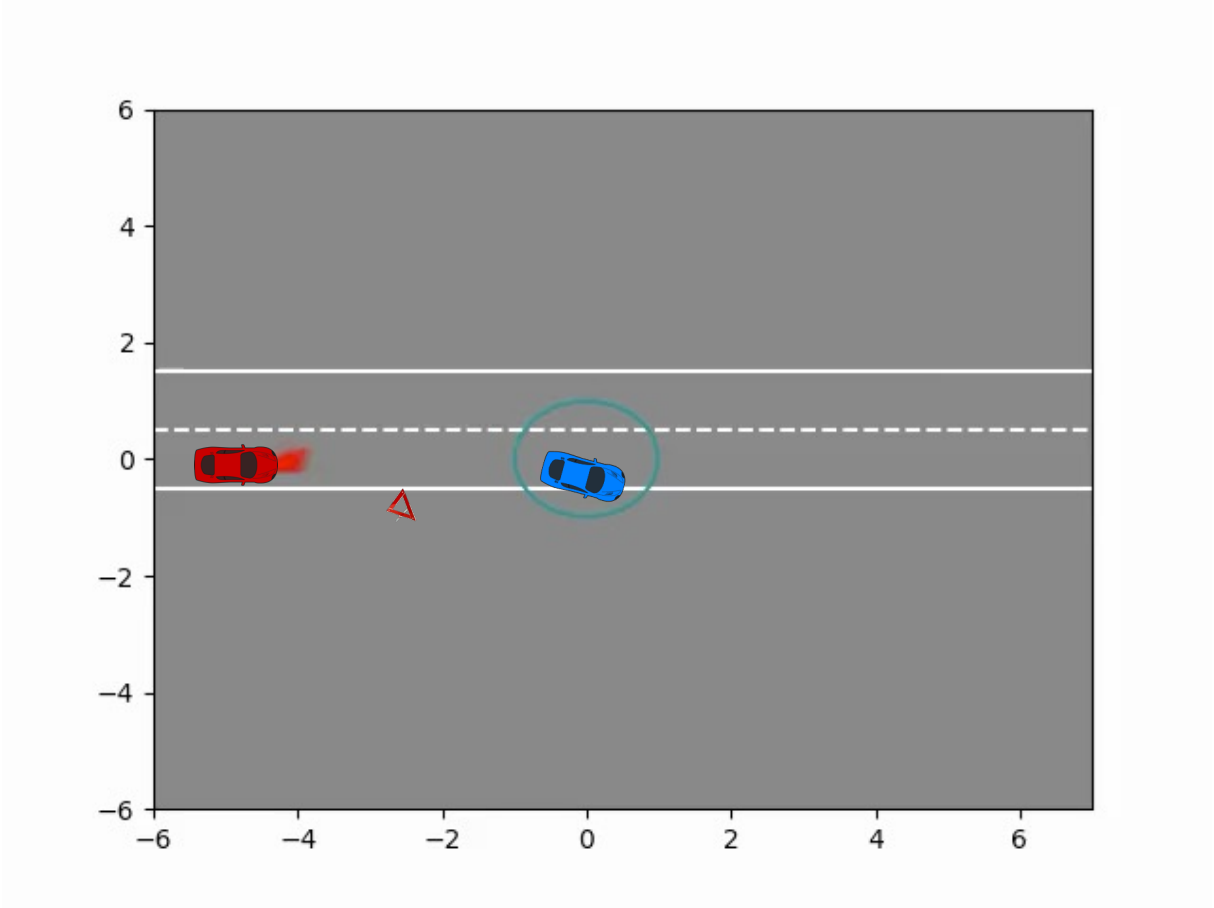
$$\theta_{k+1} = \theta_k + \alpha_k \hat{g}_k,$$

or via another gradient ascent algorithm like Adam.
8: Fit value function by regression on mean-squared error:

$$\phi_{k+1} = \arg\min_{\phi} \frac{1}{|\mathcal{D}_k| T} \sum_{\tau \in \mathcal{D}_k} \sum_{t=0}^{T} \left(V_\phi(s_t) - \hat{R}_t \right)^2,$$

typically via some gradient descent algorithm.
9: **end for**

Example



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Etienne Bührle| FZI | buehrle@fzi.de

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